Super-Eggs and Sundials

Some Examples of the Work of Piet Hein

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Man is that animal who himself draws the lines that he himself stumbles over.
Piet Hein, 1967[1]

The Danish scientist and poet Piet Hein (b. 1905) is perhaps best known for two contrasting accomplishments: on the one hand his collections of epigrams known as 'Grooks'[2], and on the other his development of the 'super-ellipse'.

To understand the latter we must first note that the ordinary ellipse is but one member of a family of curves obeying the following general formula in Cartesian coordinates:

$$\frac{x^n}{a^n} + \frac{y^n}{b^n} = 1$$

where 'a' and 'b' are arbitrary constants that represent the two semiaxes of the curve, and 'n' is any positive number.[3] When n=2 the equation is that of an ellipse with its centre at the origin of the two coordinates. This will be familiar to students of A-level mathematics.

However, it is less generally known that as n decreases from 2 towards 1 the curves become more pointed at the ends (Hein calls them 'sub-ellipses'), until when n=1 the figure is a diamond-like parallelogram (Fig.1). With n less than 1 the four sides become increasingly concave, until at n=0 they degenerate into two crossed straight lines.

If n is allowed to increase above 2 the oval develops flatter and flatter sides, becoming more and more like a rectangle.

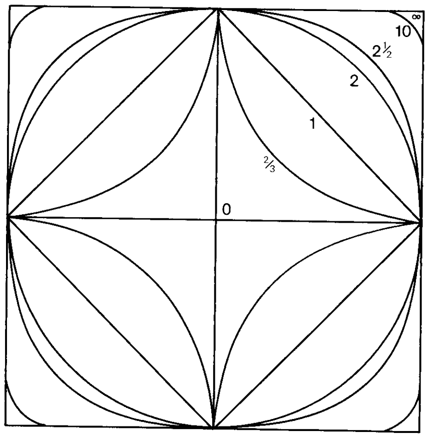

Fig.1 *Curves with a = b and n going from zero to infinity.*

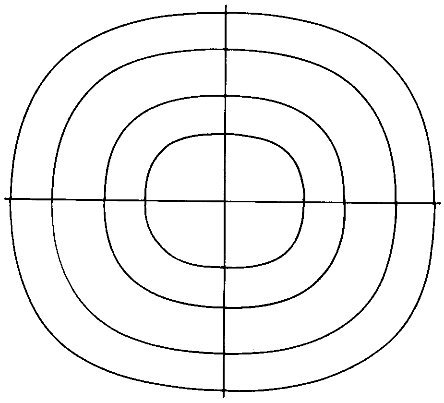

Fig.2 *Concentric superellipses with an exponent of $2\frac{1}{2}$ nest one within another.*

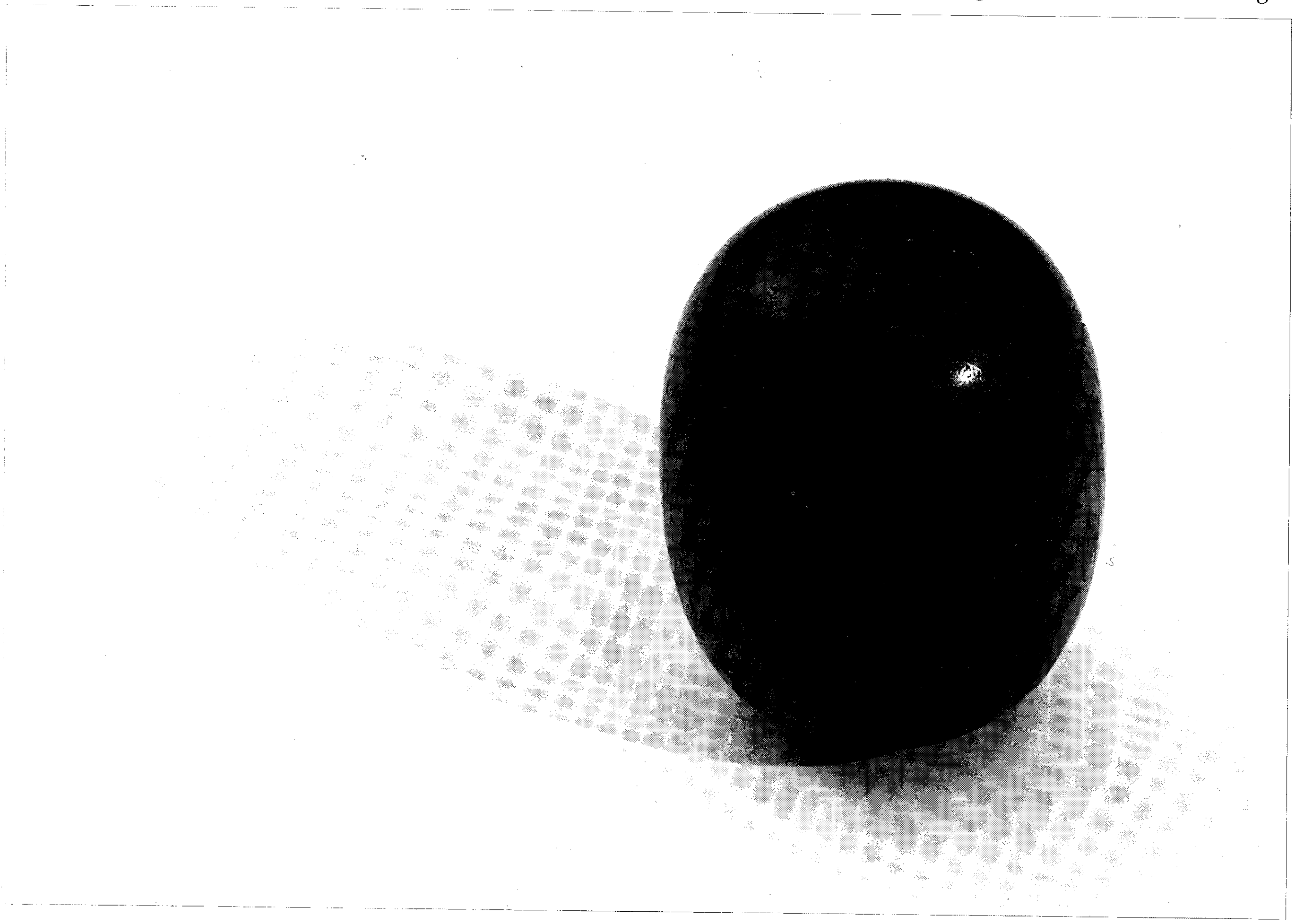

Fig.3 *A super-egg with an axial ratio of 4:3. In teak, with a major axis 8 cm long.*

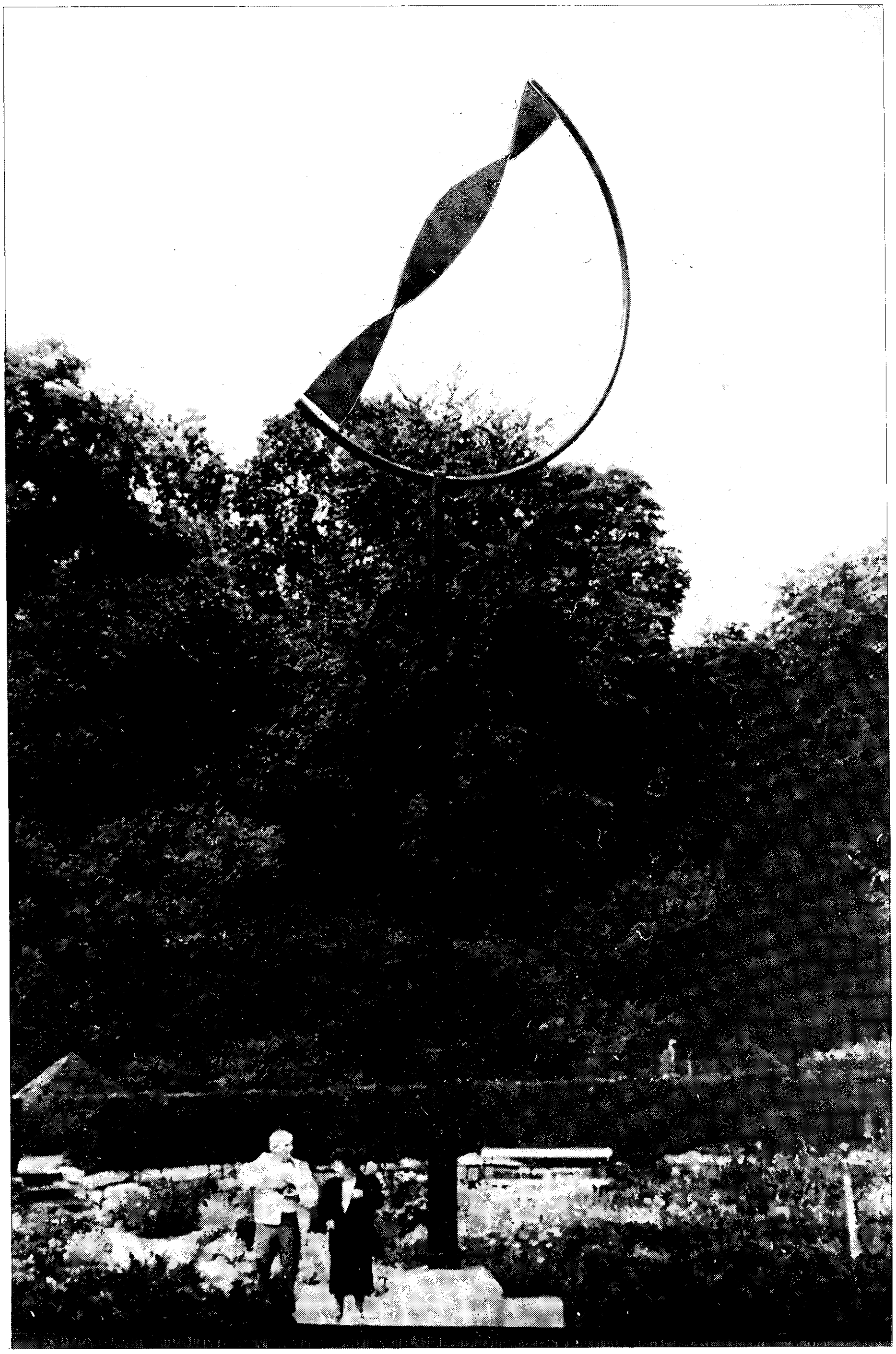

Fig.4 *The Piet Hein sundial in the grounds of Egeskov Castle, Funen, Denmark. (Photo by H Andersen)*

Indeed, that is the shape achieved when n approaches infinity (Fig.1). So here we have an infinite number of 'superellipses' — curves intermediate between the ellipse and the rectangle; or the circle and the square when a = b to give axes of equal length.

This has been known to generations of mathematicians, but not until 1960 did Piet Hein combine science with artistic intuition to realise that $n = 2\frac{1}{2}$ gave a curve that was remarkably pleasing to the eye: a satisfying blend of the ellipse and the rectangle (Fig.2). In Scandinavia, the $2\frac{1}{2}$ exponent superellipse was enthusiastically adopted for many situations requiring a curve mediating and harmonizing the right angle and the circle that have for so long dominated town planning, architecture, and interior design.[4,5]

Any of the above curves may be rotated about its x or y axis to give a three-dimensional solid. Rotating the $2\frac{1}{2}$ exponent superellipse gives the 'super-egg'[6], an object of unusual but pleasing symmetry and outline that (unlike a real egg) can stand upright on either end and be stable to small disturbances.[3]

Figure 3 shows a super-egg with an axial ratio of 4:3. It was made by cutting a metal template to the calculated curve, and then turning a piece of seasoned teak to match. When nearly finished, the object was parted-off and pressed into a cavity turned in piece of softwood to complete the shaping of the end which had been towards the chuck.

Further examples of Piet Hein's originality and ingenuity are his designs for a sundial and an extensive hedge-maze, both to be found in the beautiful gardens surrounding Egeskov Castle, near Kværndrup on the island of Funen, Denmark. Claimed to be the best-preserved Renaissance island castle in Europe, the house, gardens and veteran car museum are open daily from May to September.

The entire sundial is shown in Figure 4: with its supporting column it reaches a height of 9 metres. A closer view is given in Figure 5, from which it may be seen that gnomon and dial are combined in a single-turn helix of thin metal mounted with its long axis directed towards the celestial pole. The ends of the helix are fixed in the plane of the meridian, and so its centre (180° away) will also be in this vertical plane. It will therefore be apparent that the rays from the noon sun at the equinoxes will illuminate one half of the helix, leaving the other half in shadow. This is clear in Figure 5. However, it will also be appreciated that *both* edges (acting as gnomons) and *both* sides of the strip (acting as receiving surfaces for the shadows) are operative, although light and dark zones are interchanged on the two surfaces. Their boundary marks the time of day.

This may be clearer in Figure 6, which shows a similar helical sundial made by the author: both photographs were taken at 2 pm. As the day advances so the light/dark boundary rises at 15° of pitch per hour: knowing this length enables the 'screw' to be calibrated in hours. It is necessary to graduate only the six hours (90°) either side of 12 noon, for a summer sun leaving the upper graduated section at 6 pm will immediately give rise to a shadow boundary climbing the lower section, so that '7' for example can denote both 7 am and 7 pm. Piet Hein has chosen a left-handed helix, but a right-handed version would exhibit similar behaviour with light and dark areas interchanged.

The declination of the sun does, of course, vary in the course of the year from +23° to -23° either side of the celestial equator. Therefore the shadow boundary will not in general be exactly at right angles to the edge, requiring the hour markings to be

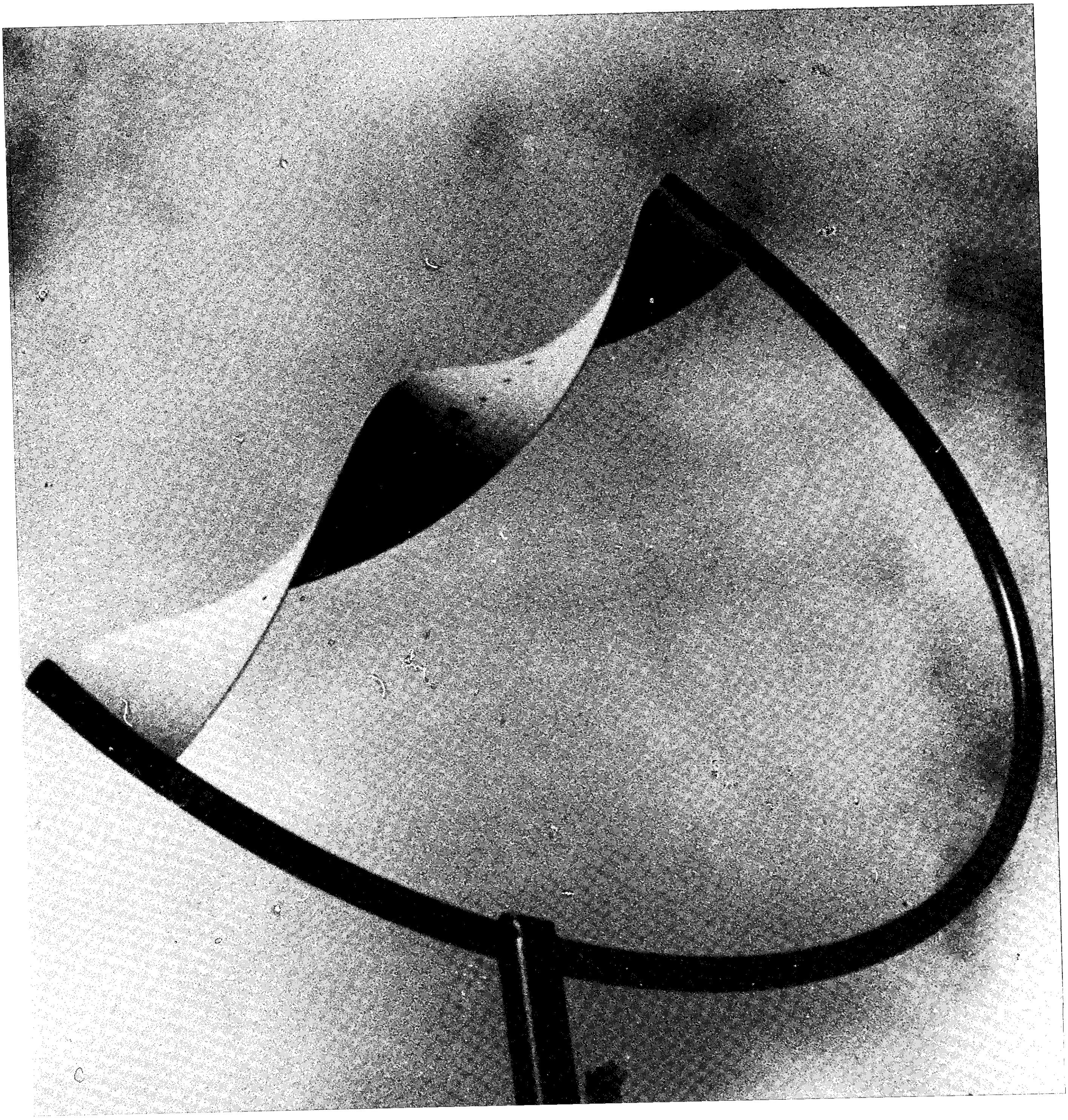

Fig.5 *Here is Time*
in heavenly grace,
Hither brought
from outer space,
Helios
uncomplicated
In a helix
concentrated.

Piet Hein

made as spots along the central axis on both sides of the strip.

If the helix is mounted on a stub axle that is a friction fit in its socket (as in Figure 6) then the entire sundial may by rotated on this axis to allow for Summer Time or the longitude correction required to give Standard Time.

The Piet Hein sundial is the simplest member of the rarely constructed family of helical dials. Ingenious as it is, the design is not particularly precise as a timeteller for the separation between the hour marks is set by the pitch of the helix, and the 'lever arm' (the distance between edge-gnomon and hour mark) is very short. More accurate members of the family increase the physical separation of a wire or rod gnomon from the shadow-receiving helical surface.

Acknowledgements

I thank Hemming Andersen for bringing the Piet Hein sundial to my attention, and for supplying photographs.